$N\pi$ Finite-Volume Energy Spectrum from $N_f = 2+1$ Lattice QCD

Sarah Skinner [1] Colin Morningstar [1] John Bulava [2] Fernando Romero-López [3]

[1]Carnegie Mellon University, Pittsburgh, PA, USA [2]Deutsches Elektronen-Synchrotron, Zeuthen, Germany [3]Massachusetts Institute of Technology, Cambridge, MA, USA

Introduction

- The Standard Model describes all basic particles and the basic forces, electroweak and strong.
- **Quantum Chromodynamics (QCD)** describes how particles behave under the strong force
- **"Chromo"** stands for the color charge (RGB) that the quarks and gluons carry
- Only quarks and gluons interact via the strong force
- The strong force is the force that holds together particles such as pions, protons, and neutrons
- Our simulation only includes **2+1** quarks, the **up (u)**, **down (d)**, and **strange (s)**

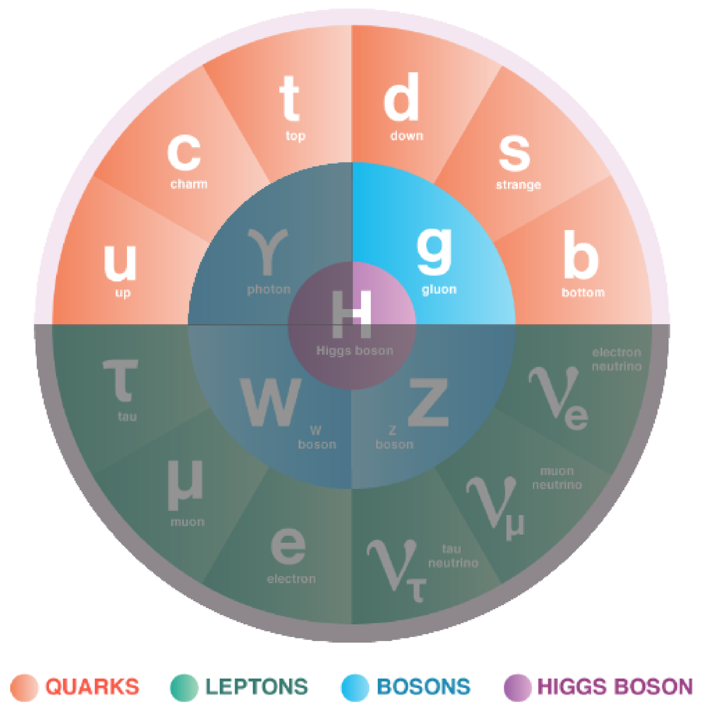

Figure 1. Particles in the Standard Model. Strong force particles are unshaded (does not include Higgs boson) [1].

$N\pi$ Scattering: A Delta Resonance

Our results focus on nucleon-pion scattering where the nucleon can be a proton or neutron. We look at all interactions where

$$N\,\pi \longrightarrow N\,\pi$$

Different variations of this interaction are called channels. One such channel contains a delta resonance, where a delta particle is created and then decays back into a nucleon pion pair.

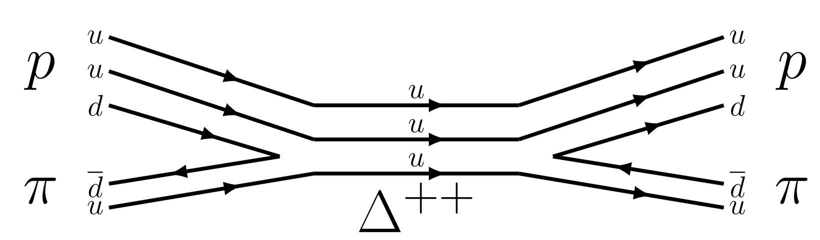

Figure 2. Proton (p) and pion (π) scattering with a delta (Δ) resonance example.

Simple Harmonic Oscillator Toy Model

To demonstrate the methodology to calculate the energy spectrum from first principles physics, we will begin with a toy model: a 1D simple harmonic oscillator (SHO).

An SHO example: think of the movement of a pendulum viewed from above.

First principle physics: Lagrangian of SHO:

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 \tag{1}$$

where x is the position of mass m, $\dot{x} = \frac{dx}{dt}$, and ω is the angular frequency. A Lagrangian describes the dynamics of the system using the energies, L = kinetic energy $-$ potential energy. t is time.

References

[1]*The standard model of particle physics*, `https://www.symmetrymagazine.org/standard-model/`.

[2]G. S. Bali, E. E. Scholz, J. Simeth, and W. Söldner (RQCD), "Lattice simulations with $N_f = 2+1$ improved Wilson fermions at a fixed strange quark mass", Phys. Rev. **D94**, 074501 (2016).

[3]M. Bruno et al., "Simulation of QCD with $N_f = 2+1$ flavors of non-perturbatively improved Wilson fermions", JHEP **02**, 043 (2015).

[4]*Juqueen*, `https://web.archive.org/web/20190118115058/https://www.fz-juelich.de/ias/jsc/EN/Expertise/Supercomputers/JUQUEEN/JUQUEEN_node.html`.

[5]*Texas advanced computing center*, `https://www.tacc.utexas.edu/systems/frontera`.

[6]M. Lüscher, `https://luscher.web.cern.ch/luscher/openQCD/`.

[7]C. Morningstar, J. Bulava, B. Fahy, J. Foley, Y. C. Jhang, K. J. Juge, D. Lenkner, and C. H. Wong, "Extended hadron and two-hadron operators of definite momentum for spectrum calculations in lattice QCD", Physical Review D **88**, `10.1103/physrevd.88.014511` (2013).

SHO Energy Spectrum from Monte Carlo

In classical mechanics, there is one path from point (x_a, t_a) to (x_b, t_b), but for quantum mechanics, a particle can take any path. In Monte Carlo, we use random numbers to simulate these possible paths.

qrd.by/sho

The quantum mechanics for a system can be characterized by the transition amplitude that determines the probability that starts at point (x_a, t_a) and ends at (x_b, t_b) by integrating over all possible paths with phase amplitude $\exp(iS/\hbar)$

$$Z(b,a) = \int_a^b \mathcal{D}x\, e^{iS/\hbar} \xrightarrow{t\to -i\tau} \int_a^b \mathcal{D}x\, e^{-S/\hbar} \tag{2}$$

where S is the action $S = \int_{t_a}^{t_b} L\, dt$. Under a Wick rotation of time ($t \to -i\tau$), this phase turns into a weight, i.e. each path has a probability $\exp(-S/\hbar)$ to occur. To see how different paths contribute to the transition probability, several paths are mapped out on a lattice in the τ domain in Figure 3.

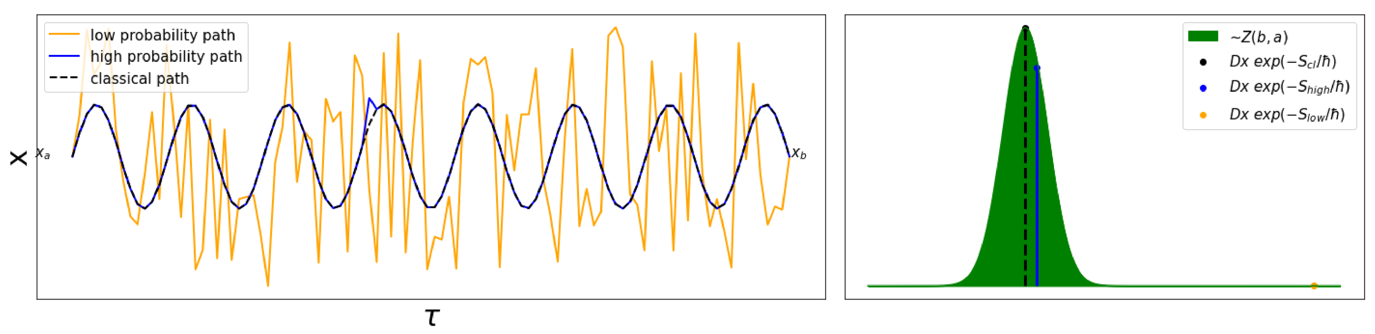

Figure 3. A few paths and their contribution to the transition amplitude. Available at qrd.by/sho.

To capture the quantum physics, we want to generate paths that are primarily in the peak of the transition amplitude, so we use the Metropolis-Hastings method to choose the paths/configurations. A few configurations are shown out in Figure 4.

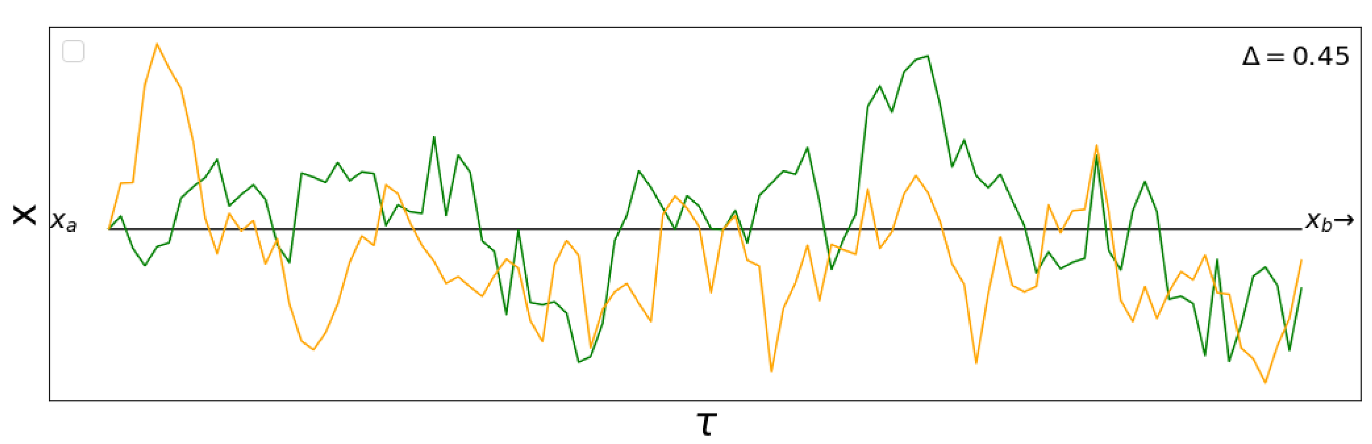

Figure 4. Configurations computed using the Metropolis-Hastings method. Available at qrd.by/sho.

Using these configurations, we can calculate correlation functions, $\langle\phi_0|x(\tau)x(0)|\phi_0\rangle$. These correlations functions can be related to the energies E_n of the system using spectral analysis with overlap amplitude A_n.

$$\langle\phi_0|x(\tau)x(0)|\phi_0\rangle = \sum_{n=0}^{\infty} A_n^2 \exp\left(\frac{-(E_n - E_0)\tau}{\hbar}\right) \tag{3}$$

Using this relation, we can fit to the lowest lying energy spectrum.

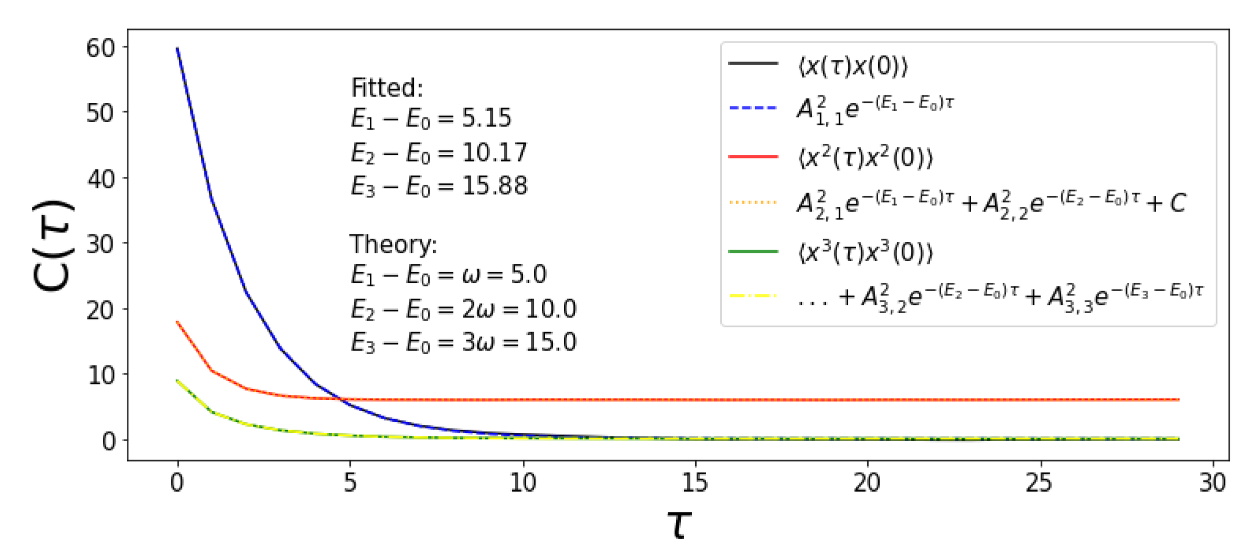

Figure 5. Several fitted correlation functions produced from SHO Example repository (qrd.by/sho). The fit results are compared to analytical calculations. $\hbar = c = 1$

Lattice QCD

QCD Lagrangian density ($L = \int d^3x\, \mathcal{L}$):

$$\mathcal{L}[\psi,\overline{\psi},\mathcal{A}] = \sum_{f=1}^{N_f} \overline{\psi}_{a\alpha}^{(f)} (i\gamma^{\mu}_{\alpha\beta}\mathcal{D}_{\mu ab} - m^{(f)}\delta_{\alpha\beta}\delta_{ab})\psi_{a\alpha}^{(f)} - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}, \tag{4}$$

$$\mathcal{D}_\mu = \partial_\mu + ig\mathcal{A}_\mu;\ G_{\mu\nu} = -\frac{i}{g}[\mathcal{D}_\mu, \mathcal{D}_\nu];\ \mathcal{A}_\mu = \mathcal{A}^a_\mu\frac{\lambda_a}{2} \tag{5}$$

- $\psi, \overline{\psi}$ – fermionic quark fields , Dirac spinors with mass m and flavor f
- $\mathcal{A}_\mu$ – gluon fields , non-abelian, SU(3) symmetry described by Gell-Mann matrices λ
- γ^μ – Dirac gamma matrices
- g – coupling strength
- fermionic color indices $a, b = 1, 2, 3$
- gluonic color indices $a = 1, 2, ...8$
- Dirac indices $\alpha, \beta = 1, 2, 3, 4$
- Minkowski space-time indices $\mu, \nu = 1, 2, 3, 4$ – x,y,z,t

Changes from SHO to QCD:

- QCD is 4D - 3 spacial dimensions and 1 time dimension
- QCD is a gauge theory, adds constraints to the degrees of freedom
- for every flavor of quark there are 2 corresponding 12-vector fields $\psi, \overline{\psi}$
- there are 8 gluon fields

To retrieve the energy spectrum, we use hadronic annihilation operators in our time-ordered 2-point correlator in natural units ($\hbar = c = 1$):

$$C_{ij}(t) = \langle 0|T\mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\overline{\mathcal{O}}_j|0\rangle e^{-(E_n - E_0)t} \tag{6}$$

where hadronic operators can represent the individual particles N,π, or the combined $N\pi$ system.

Results and Conclusions: $N\pi$ Energy Spectrum

Parameters of the D200 ensemble produced by the Coordinated Lattice Simulation Group can be found in Refs. [2, 3]. Configurations were calculated on JUQUEEN [4], and correlators on Frontera [5]. `openQCD` was used for many calculations [6].

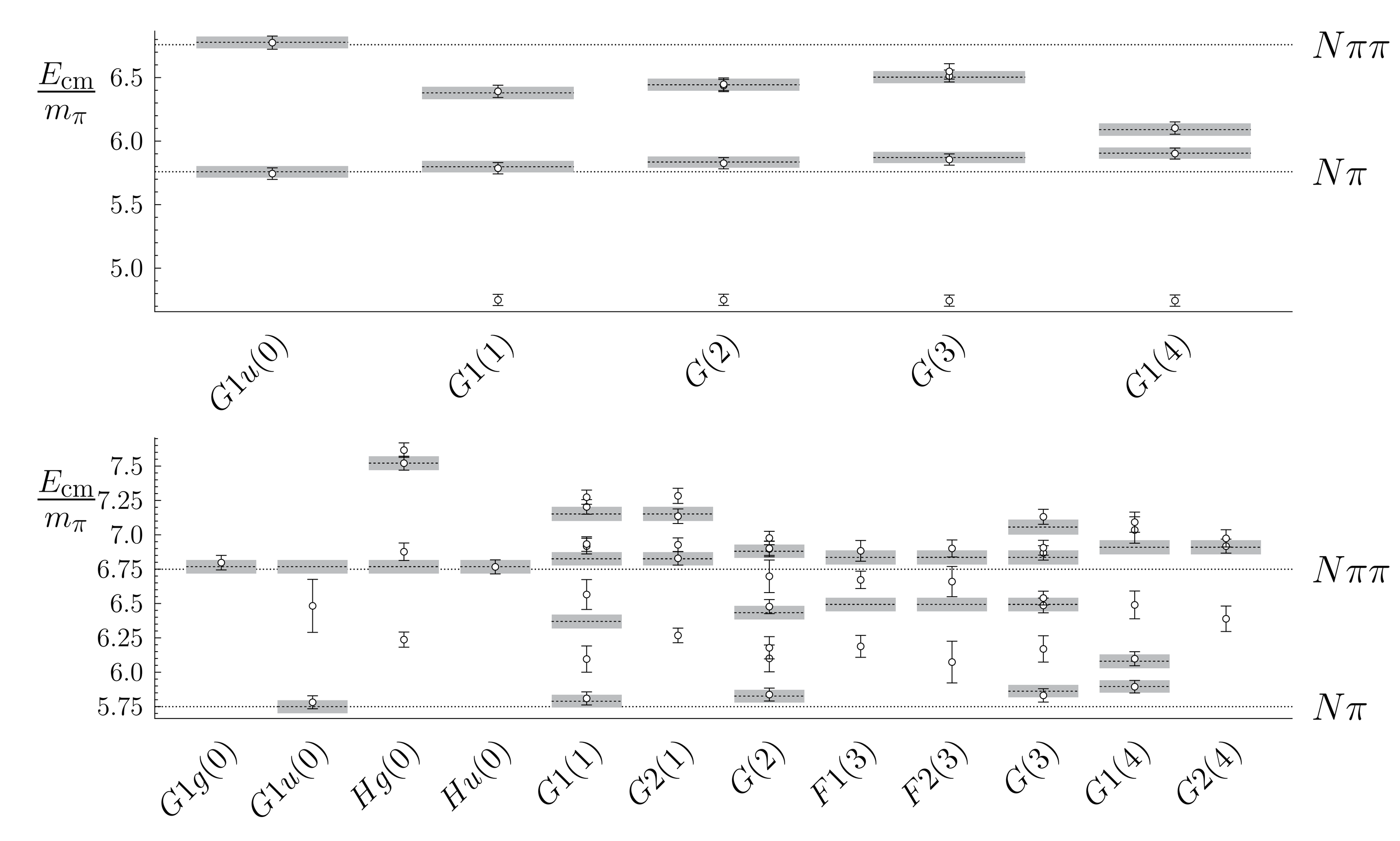

Figure 6. Top: $I = 1/2$. Bottom $I = 3/2$. The notation along the horizontal axis is $\Lambda(\mathbf{P}^2)$, where $\mathbf{P}^2$ is the total momentum squared and Λ is the irrep of little group $\mathbf{P}$ [7]. Dashed lines indicate the limits of the elastic region. Solid lines and shaded regions indicate the non-interacting levels and their errors.

The $N\pi$ channels that we study here are also known as the roper and delta resonance channels. We can see evidence of resonances when the energy spectrum differs from the non-interacting spectrum. Though we don't see this behavior in the $I = 1/2$ channel, we do see evidence of delta resonance in $I = 3/2$ channel. Using this data, we can investigate the delta resonance, which is needed information for the Deep Underground Neutrino Experiment (www.dunescience.org).

Research funded by the *National Science Foundation*

sarahski@andrew.cmu.edu · qrd.by/xinu